

THE ELEMENTARY $p(p,p'\pi^+)n$ REACTION

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Abstract

A detailed study of the elementary $p(p,p'\pi^+)n$ reaction is presented using the delta isobar model. In this model, in the first step one of the two protons in the initial state gets excited to Δ . This, in the second step, decays into a nucleon and a pion. For the $pp \rightarrow N\Delta$ step the parametrized form of the DWBA t-matrix of Jain and Santra, which reproduces most of the available data on $pp \rightarrow n\Delta^{++}$, is used. The cross-sections studied include the outgoing proton momentum spectra in coincidence with the pion, the outgoing pion momentum spectra and the integrated total cross-section. We find that all the calculated numbers are in good agreement with the corresponding measured cross sections.

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1 Introduction

In past various authors [3, 4], including two of the present authors (Jain and Santra), have analysed theoretically the data on the $pp \rightarrow n\Delta^{++}$ reaction to extract the potential for $pp \rightarrow N\Delta$ transition. In them, the calculations of Jain et al. [4] were done in the DWBA and those of Dmitriev [3] were done in the PWBA. They concluded that the spin averaged data on the $pp \rightarrow N\Delta$ reaction can be reproduced very well by a one pion-exchange potential with the length parameter Λ_π around 1-1.2 GeV/c in DWBA and around 650 MeV/c in the PWBA. The difference in the two values of Λ_π is due to distortion effects. In fact, subsequently, when Jain et al. parametrized their DWBA t-matrix [6], they found that the imaginary part of this t-matrix is very weak and the real part resembles to a great extent the one pion-exchange potential, with Λ_π reduced to around 650 MeV/c.

The experimental data which above studies used were somewhat inclusive [2, 5]. They were deduced from the $pp \rightarrow np/\pi^+$ reaction data which did not have the complete exclusive kinematics. The delta was identified in them by seeing a bump in the missing mass spectrum. A kinematically complete data set, however, exists on the $pp \rightarrow p/\pi^+n$ reaction at 800 MeV beam energy from LAMPF due to Hancock et al.[1]. They are a good coincidence data, and, thus, provide an excellent opportunity to test in detail the correctness of the $pp \rightarrow n\Delta^{++}$ DWBA t-matrix developed by two of us earlier [6]. In the present paper we analyse the LAMPF data using this t-matrix. This includes the analysis of the various proton and pion energy spectra measured in coincidence and the total integrated cross section for the $pp \rightarrow p/\pi^+n$ reaction. We assume that the $pp \rightarrow p/\pi^+n$ reaction proceeds in two steps. In the first step, one of the protons in the entrance channel gets converted to Δ , and in the second step this delta decays into a pion and a nucleon. The transition matrix for the $pp \rightarrow \Delta N$ step is taken to be the DWBA t-matrix mentioned above. The decay of the delta is described by the pseudovector non-relativistic Lagrangian,

$$L_{\pi N\Delta} = i \frac{f_\pi^*}{m_\pi} (\mathbf{S} \cdot \boldsymbol{\kappa}_\pi) (\mathbf{T} \cdot \boldsymbol{\phi}), \quad (1)$$

where f_π^* is the coupling constant at the $\pi N\Delta$ vertex. \mathbf{S} and \mathbf{T} are the spin and isospin transition operators, respectively. This framework for the $pp \rightarrow p\pi^+n$ reaction includes in a certain way the final state interaction [FSI]

amongst $p\pi^+n$ in the final state. The FSI consists of the interaction between p and π^+ and between the $p\pi^+$ pair and the recoiling neutron. The dominant effect of the interaction between p and π^+ is to produce the Δ^{++} resonance. This is explicitly included in our framework. The interaction between $p\pi^+$ and the neutron in our framework is approximated by that between the Δ^{++} and the neutron. A recent work by Jain and Kundu [7] on the delta decay in nuclear medium suggests that this approximation is reasonably good.

The $pp \rightarrow np'\pi^+$ process has also been worked out in the literature by Engel et. al [8]. However, these calculations use plane waves for the continuum particles. Thus, unlike our work, this work does not include the effect of distortions in the entrance and the exit channels.

Inclusion/omission of rho-exchange in the description of the $pp \rightarrow n\Delta^{++}$ reaction has been the topic of much debate in the literature. The general conclusion is that the spin averaged data on the $pp \rightarrow \Delta^{++}n$ reaction are well reproduced by one pion-exchange potential only [9, 3, 4, 10]. Any attempt to include the rho-exchange worsens the agreement with the experiments, and yield unsatisfactory results. In this context it is also interesting to see the work of Jain et al. [11] which discusses the relative importance of rho-exchange in $p(n,p)n$ and $p(p,n)\Delta^{++}$ reactions. They conclude that, while it is absolutely essential to include the rho-exchange in the description of the $p(n,p)n$ reaction, the rho-exchange is not required for accounting the $p(p,n)\Delta^{++}$ data. This study deals with the spin averaged cross sections. A recent theoretical study on the microscopic structure of the $\rho N\Delta$ vertex by Haider et al. [12] supports this conclusion. They find that the microscopically calculated value of the $f_{\rho N\Delta}$ coupling constant is much smaller than what is normally assumed. The measured spin averaged cross sections on nuclei in charge exchange reactions are also reproduced with only a pion exchange [13]. It is, however, true that the measurements of Prout et al. [14] with a polarized proton beam on nuclei, and earlier by Ellegaard et al. [15] do show a large transverse part. But, as shown by V.F.Dmitriev [13] and Sams et al. [16], large transverse contribution can also arise from the distortion of the continuum particles. All these discussions thus suggests that, at best, the role of rho-exchange in the charge-exchange reaction in the delta region is controversial. The spin averaged cross sections do not need it, the spin transfer measurements show some indications for it. Since the present work deals with the spin averaged cross sections, our use of one pion-exchange is consistent with other work in this field.

In section 2 we write the formalism for the $pp \rightarrow np'\pi^+$ process. Section 3 gives calculated cross sections for the proton and pion energy spectra at 800 MeV beam energy and the total cross section from 500 MeV to 2 GeV. These results are compared with the available experimental cross sections. A good agreement is obtained.

2 Formalism

The cross-section for the $pp \rightarrow np'\pi^+$ process is given by

$$d\sigma = \langle |t_{pp \rightarrow p'\pi^+n}|^2 \rangle [PS], \quad (2)$$

where, the angular brackets denote the sum and average over the spins in the initial and final states, respectively. $[PS]$ is the factor associated with the phase-space and the beam current. For the proton and pion detected in coincidence in the final state, in the lab. frame it is given by,

$$[PS] = \frac{m_p^2 m_n k_p'^2 k_\pi^3}{2(2\pi)^5 k_p E_p' k_\pi^2 (E_i - E_p') - E_\pi |(\mathbf{k}_p - \mathbf{k}_p') \cdot \mathbf{k}_\pi|} d\Omega_p' d\Omega_\pi dk_p'. \quad (3)$$

$t_{pp \rightarrow p'\pi^+n}$ is the t-matrix for the $pp \rightarrow p'\pi^+n$ process. It consists of two parts: one corresponding to the excitation of the proton in the initial state to Δ^{++} and another corresponding to its excitation to Δ^+ [Figure 1]. That is

$$t_{pp \rightarrow p'\pi^+n} = t^{\Delta^{++}} + t^{\Delta^+}. \quad (4)$$

Furthermore, because of the antisymmetrization of the protons, each t-matrix in turn consists of two terms, one corresponding to the excitation of the beam proton and another corresponding to the excitation of the target proton. We call them “direct” and “exchange” terms, respectively.

Putting every thing together, we get

$$\begin{aligned} t_{pp \rightarrow NN\pi} &= \sum_{\Delta} \langle N\pi | \mathbf{S} \cdot \boldsymbol{\kappa}_\pi \mathbf{T} \cdot \boldsymbol{\phi}_\pi | \Delta \rangle \\ &\times G_{\Delta} \langle t_{pp \rightarrow N\Delta} \rangle, \end{aligned} \quad (5)$$

where N represents a proton or a neutron in the final state corresponding to the decay of $\Delta^{++} \rightarrow \pi^+p$ and $\Delta^+ \rightarrow \pi^+n$, respectively. Δ stands for a Δ^{++}

or Δ^+ excitation in the intermediate state. κ_π at the Δ -decay vertex is the outgoing pion momentum in the πN centre-of-mass. It is given by,

$$\kappa_\pi(\mu^2, m_\pi^2) = [(\mu^2 + m^2 - m_\pi^2)^2 / 4\mu^2 - m^2]^{1/2}. \quad (6)$$

This relation reflects the restrictions on the available phase space for the decay of a delta of mass μ into an on-shell pion of mass m_π (=140 MeV) and a nucleon. Since the final outgoing pion is on-shell, the $\Delta N \pi$ vertex does not contain the usual form factor F^* . G_Δ in equation 5 is the delta propagator. Its form is taken as,

$$G_\Delta = \frac{2m_\Delta}{\mu^2 - m_\Delta^2 + i\Gamma_\Delta m_\Delta}, \quad (7)$$

where, m_Δ (=1232 MeV) and Γ_Δ are the resonance parameters associated with a free Δ . The free width, Γ_Δ , depends upon the invariant mass and is written as,

$$\Gamma_\Delta = \Gamma_0 \left[\frac{k(\mu^2, m_\pi^2)}{k(m_\Delta^2, m_\pi^2)} \right]^3 \frac{k^2(m_\Delta^2, m_\pi^2) + \gamma^2}{k^2(\mu^2, m_\pi^2) + \gamma^2}, \quad (8)$$

with Γ_0 =120 MeV and γ =200 MeV. μ is the invariant mass of the $N\pi^+$ system and is given by,

$$\mu^2 = (E_N + E_\pi)^2 - (\mathbf{k}_N + \mathbf{k}_\pi)^2. \quad (9)$$

$t_{pp \rightarrow N\Delta}$ is the DWBA t-matrix for the $pp \rightarrow N\Delta$ transition. Following Jain and Santra [4], it is given by

$$t_{pp \rightarrow N\Delta} = (\chi_{\mathbf{k}_f}^-, < n\Delta^{++} | v_\pi | \{pp\} >, \chi_{\mathbf{k}_i}^+), \quad (10)$$

where curly brackets around pp represent the antisymmetrization of the pp wave function. v_π is the one pion-exchange potential for $pp \rightarrow N\Delta$ transition. χ s are the distorted waves. They describe the elastic scattering of the pp and the $n\Delta$ systems. Jain and Santra [4] have evaluated equation 10 using eikonal approximation for χ s. With Λ_π =1 GeV/c at both the πNN and $\pi N\Delta$ vertices, they found that this t-matrix reproduces the available experimental data on this reaction over a large energy range very well.

Jain and Santra also found that their DWBA t-matrix can be easily parametrized [6]. The parametrized t-matrix is complex, but its imaginary

part is very weak. The real part resembles very much with the one pion-exchange potential with its length parameter, Λ_π , reduced to around 600-700 MeV/c. For the present calculations, instead of repeating the full calculation of the t-matrix, we have used the parametrized form, i.e.

$$t_{pp \rightarrow N\Delta} \approx v_\pi^{pp \rightarrow N\Delta}(\Lambda_\pi = 650 \text{ MeV}/c) = -\frac{ff^*}{m_\pi^2} FF^* \frac{\mathbf{S}^+ \cdot \mathbf{q} \sigma \cdot \mathbf{q}}{m_\pi^2 + q^2 - \omega^2} \mathbf{T}^+ \cdot \boldsymbol{\tau}, \quad (11)$$

where f and f^* at the πNN and $\pi N\Delta$ vertices are 1.008 and 2.156 respectively [17]. \mathbf{q} is the momentum transfer in the pion-nucleon rest frame. Since the exchanged pion is virtual, it is not straight forward to define this momentum quite unambiguously. For the $\pi N\Delta$ vertex we use the following Galilian invariant form,

$$\mathbf{q} = \mathbf{k}_p - \mathbf{k}_\Delta [= (\mathbf{k}_N + \mathbf{k}_\pi)] - \frac{\omega \mathbf{k}_\Delta}{E_\Delta}, \quad (12)$$

where, ω is the energy transfer in exciting the Δ . At the πNN vertex we replace

$$\mathbf{q}^2 \rightarrow -t, \quad (13)$$

where t is the four momentum squared.

3 Results and Discussion

Using the above formalism we calculate the exclusive proton momentum spectra, the outgoing pion momentum spectra and the integrated total $p(p, p' \pi^+)n$ cross-section.

As the detailed measurements for the $p(p, p' \pi^+)n$ process exist at 800 MeV beam energy, we first calculate the differential cross-sections at this energy. In figure 2, we plot the calculated as well as the measured [1] exclusive proton momentum spectra for the proton and the pion angles of 14.5° and -21° degrees, respectively. These angles correspond to the delta going at 0° . The figure has got four calculated curves. The short-dashed and dot-dashed curves correspond to Δ^{++} and Δ^+ contributions (including both, the “direct” as well as “exchange” diagrams), respectively. The solid curve is the coherent sum of these two contributions. We find that this curve agrees well

with the measured cross sections. We also note that the main contribution to the solid curve comes from the Δ^{++} diagram. The Δ^+ contributes only to the extent of 5-10 %.

To show the contribution of the “exchange” diagram, in fig. 2 we also show (by long-dashed curve) the cross section for the Δ^{++} diagram using only the “direct” term. Comparing this with the short-dashed curve, which includes both the direct and exchange diagrams, we find that the contribution of the exchange term is around 15-20%.

In figure 3, we show the proton spectrum for another set of proton and pion angles. This pair of angles also correspond to the delta going at 0^0 . The outgoing proton and pion angles are 14.5^0 and -42^0 , respectively. All the curves have the same meaning as those in figure 2. Here too the calculated proton spectrum is in good accord with the measured spectrum. Other observations also remain same as in fig. 2.

In figure 4 we show the double differential cross-section as a function of the outgoing pion momentum. The proton angles are integrated. Experimentally such measurements exist for 800 MeV beam energy and the pion detected at 20^0 [18]. In this figure we have 3 curves alongwith the experimental data. The dash and dash-dot curves correspond separately to the Δ^{++} and Δ^+ diagrams, respectively. The solid curve is calculated including both the diagrams. All the curves include the direct as well as exchange diagrams. Excluding the peak in the measured cross sections around 550 MeV, the solid curve is in overall accord with the measured cross sections. Relative contributions of the Δ^+ and Δ^{++} to the cross sections are at the same level as in the earlier curves. The peak around 550 MeV, as kinematic considerations suggest, may arise from the resonance structure between neutron and proton in the final state.

Finally in figure 5 we present the calculated total integrated cross section as a function of the beam energy from threshold to 2 GeV. Since, as seen from the results in figures 2 - 4, the contribution of the Δ^+ is only at the level of 10%, we give the calculated results for the Δ^{++} only. The calculated results include both the direct and the exchange contributions. We find an excellent agreement between the calculated and measured cross-sections [19].

4 Conclusions

In conclusion, the findings of this paper can be summarized as :

1. Experimentally measured exclusive proton momentum spectra, the pion momentum spectrum and the total integrated cross sections over a large energy range can be reproduced well with one-pion exchange potential for the delta excitation in the intermediate state;
2. the contribution of the Δ^{++} dominates. Δ^+ contributes only to the extent of 5-10%, and
3. the effect of the exchange process is to bring down the cross-section. Its contribution, however, is only at the level 10-20%.

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Figure Captions

1. The direct and exchange diagrams for the Δ excitation.
2. The outgoing proton momentum spectrum in coincidence with the pion. $T_p=800$ MeV. $\theta'_p=14.5^\circ$ and $\theta_\pi=-21^\circ$. The experimental points are from [1]. The long-dashed curve is calculated using the direct Δ^{++} diagram and the short-dashed curve includes both the direct and the exchange Δ^{++} diagrams. The solid curve is calculated using both the Δ^{++} and Δ^+ diagrams added coherently. The dash-dot curve is the Δ^+ contribution multiplied by 5. $\Lambda_\pi=650$ MeV/c.
3. Same as figure 2 with $\theta'_p=14.5^\circ$ and $\theta_\pi=-42^\circ$. Experimental points are from [1]. All the curves have the same meaning as in figure 2. $\Lambda_\pi=650$ MeV/c.
4. The outgoing pion momentum spectra for the $p(p,p'\pi^+)n$ reaction at $T_p=800$ MeV. $\theta_\pi=20^\circ$. The experimental points are from [18]. The solid curve is calculated using both the Δ^{++} and Δ^+ diagrams added coherently. The short-dashed and dot-dashed curves show separately the contribution due to Δ^{++} and Δ^+ , respectively. $\Lambda_\pi=650$ MeV/c.
5. Total cross-section for the $p(p,p'\pi^+)n$ reaction. The calculated curve includes both the direct and exchange Δ^{++} excitation diagrams. $\Lambda_\pi=650$ MeV/c. The experimental points are from [19].









